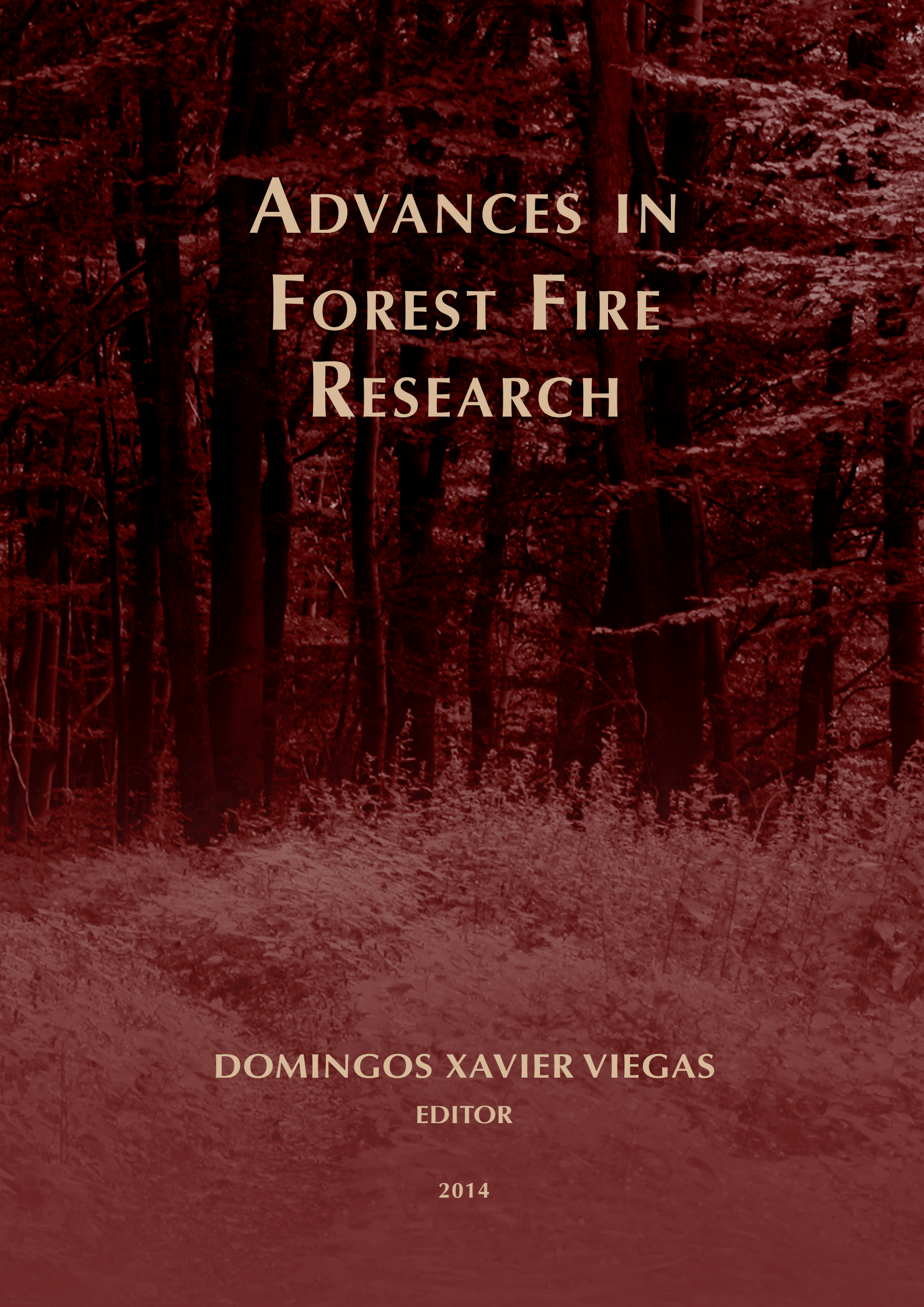

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Model reduction approach for wildfire multi-scenario analysis

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Abstract
Wildfire models have been widely applied to the prediction of fire front evolution, in order to obtain useful information for evacuation plans and fire management. A major difficulty in treating wildland fires is related to the complexity of the phenomena that are involved. In addition, it is difficult to obtain accurate input data for the models, especially in the case of on-going fire events. In fact, wind and weather data, available from meteorological stations, fuel characteristics and orographic characteristics data are often inaccurate. Furthermore, some of these data may change during the fire event, so a prediction is necessary. Results obtained from models are therefore affected by errors. Probabilistic approaches are useful in order to overcome some of these problems, but this requires the use of suitable models in order to perform large number of simulations. Over the last decades, empirical and physical based models have been proposed. Physical based models provide detailed results but requires higher computational cost than empirical models. In order to use physical based models for risk analysis and multi-scenario analysis it is necessary to reduce their computational time, which can be achieved through model reduction techniques.
In this paper, Proper Orthogonal Decomposition technique (POD) is applied to the reduction of a physical model. A simple one-dimensional physical model has been selected in order to test the approach. This model is based on conservation equations and it is built by setting some of the parameters through empirical data collected during field fire experiments. In this first work, slope and wind contribution are not considered. POD permits to extract the spatial basis of the problem and capture the main features of the system with reduced requirement of computational resources. A comparison shows that the results of the reduced model are close to that of the full model, but the computational time for solving the energy equation is reduced to about 10% of that required by the full model.

Keywords*: wildfire, reduced model, Proper Orthogonal Decomposition, fast physical model*

1. Introduction

Wildfires are complex natural phenomena that threaten both human lives and infrastructures and that have caused various disasters in the last years. Wildfire are particularly destructive and difficult to manage if proper and effective fire fighting actions are not taken in time. For this reason the prediction of wildfire evolution and propagation plays a key role and can greatly help in increasing the efficiency of fire-fighting operations. In the last decades several mathematical models have been developed to study the behavior of wildfires. The models available in the literature can be divided in three groups: theoretical, empirical and semi-empirical (Pastor *et al.* 2003). Theoretical model are also called physics-based model (Mell *et al.* 2007). In the literature various software based on physical models exist, including FIRESTAR, FIRETEC, FIRELES, WFDS (Morvan 2011). Physical models of wildfires are based on the conservation equations that describe mass momentum and energy conservation, radiative heat transfer, reaction and diffusion. Such an approach leads to complex

mathematical models involving a system of coupled, non-linear partial differential equations. As a consequence, the prediction of wildfire propagation through physical models require large computational times and relevant hardware resources (Sullivan 2009). Furthermore, wildfire modeling involves large computational domains and a wide range of time scales which makes physical models not suitable as operational tools for real-time calculations. Empirical models are developed correlating experimental data while semi-empirical model are based on energy conservation but do not distinguish the different modes of heat transfer (Mell *et al.* 2007). Typical examples of empirical models are those one proposed by Rothermel (1972) and McArthur (1965) which have been largely employed because of their simple mathematical structure and the capability of predicting key quantities such as the rate of spread (ROS). Empirical models are typically algebraic and very cheap from a computational point of view. For these reasons they represent a valuable operational tool (Mandela *et al.* 2012). However, empirical models treats the interaction between the physical phenomena in a oversimplified manner. This is the case, for example, of wind-wildfire interaction which is often predetermined and does not account of buoyancy and turbulence fluctuations. Furthermore, empirical models cannot predict key local quantities in a wildfire, such as temperature and heat flux. Empirical models are widely used in order to predict the fire spread because they give predictions with a computational cost that is lower and lower than the event evolution. On the other hand these models in some condition are not able to give accurate results.
In this framework, with the aim to reduce the computational cost of physical models, in this paper an approach of modeling based on proper orthogonal decomposition (POD) has been proposed. It is the first time that POD is applied to a wildfire prediction model. Here a one dimensional model for the investigation of fire propagation over a flat ground is proposed. The model is calibrated using a set of experimental measurements gathered during field fire experiments. Then, POD technique is introduced to derive a reduced order model that allows to investigate the fire phenomenon with a reduced computational effort. Finally, the capabilities of the POD model are assessed and a parametric analysis is carried out. To summarize this paper constitute the first work on the use of model order reduction technique for wildfire analysis.

2. Experiments and Models description

2.1. Collection of experimental data

Fire behaviour was assessed in four field fire experiments (Figure 1) in North-West Italy (Ascoli *et al.* 2013) carried out on a flat terrain in grassland fuels dominated by *Moliniaarundinacea* Shrank. Fire experiments were conducted under moderate weather at one burnday in 2009 during the winter dry season when grass fuel is fully cured.
Fuel characteristics were measured in all fire experiment sites before burning. At each site, fuels were harvested in six 1 m^2 quadrates and dried in the laboratory to determine the fuel load. Fuels were entirely constituted by dead (fully cured grasses) fine fuels (< 6 mm in diameter). Fuel bed depth and cover were measured every 0.5 m along six linear transects (length =10 m) at each fire site. Flammability parameters (surface area to volume ratio, moisture of extinction, heat content) were derived from published values for similar grass fuels (FCCS Inferred variables)[1].Fuel cover was 100% at all sites while mean fuel bed depth was 14 cm. Fuel load and bulk density ranged between 4.29to 5.50 t ha^{-1}, and 2.2 to 7.1 kg/m^3, respectively. Dead fine fuel moisture was assessed at the time of each fire experiment by collecting five samples of grass leaves (50 g of fresh weight each). Fresh samples were weighed in the field using a portable scale, and then dried in laboratory. Fuel moisture (dry weight basis) ranged between 11-19%. In order to let the fire front reach a pseudo-steady state, each fire

[1]http://www.fs.fed.us/pnw/fera/fccs/inferred_variables/table2_metric/table2_metric.htm

experiment was ignited upwind by line ignitions of 25 m in length, and the fire was allowed to spread for 50 m before being suppressed along a fuel break. During each experiment, fire behaviour was assessed at a microplot scale (Fernandes *et al.* 2001; Vacchiano *et al*. 2014) by measuring the arrival time of the fire front at the vertices of a triangle and computing the rate of spread according to the trigonometric method of Simard *et al*. (1984). At each fire experiment site, 8 equilateral triangles (10 m side) were visualized using 2 m rods placed at each triangle vertex. The time of arrival of the fire front was measured using K-Type thermocouples (0.4 mm in diameter) positioned at each triangle vertex within the grass fuel few centimeters above the soil (< 10 cm) and connected to a data-logger buried one meter apart. This allowed to measure flame temperature every second during all combustion phases. Flame height was assessed by four observers which used as a reference scale the increment markers painted on each rod (0.5 m).
Air temperature and moisture, and wind speed and direction were assessed every 30 seconds by two weather stations positioned at a height of 2 m upwind to the experimental plot, so as to couple weather data to fire behaviour observations. In a few cases, marked changes in wind direction occurred during the burn. However, the microplot approach allowed to identify backfire, flank fire and head fire phases. Time since last rain was 19 days, air temperature and moisture, and wind speed ranged between20 to 27%, 19 to 25°C, and 2.8 to 7.1 km h^{-1}, respectively.

Figure 1. Field fire experiment

In total, 32 rate of spread observations and 32 time-temperature profiles were collected, each associated with fuel characteristics (load; bulk density; moisture), environmental conditions (slope; wind speed) fire behaviour (rate of spread; back, flank or head fire phase) and effects (fuel consumption). Rate of spread ranged between 0.8 to 14.2 m min^{-1}. Only observations corresponding with negligible wind velocity and in flat ground have been considered for the model development. Fuel consumption was assessed soon after the fire by collecting remaining charred fuels in six 1 m^2 quadrats and ranged between 75 and 90% of the pre-fire mean fuel load. Maximum flame temperature ranged between 244°C and 733°C. Average residence time above 60°C and 300°C were 183 and 21 seconds, respectively.

2.2. Model description

In this paper a one-dimensional model was developed in order to investigate the fire propagation in a grass-shrub flat and the corresponding temperature time evolution. The computational domain considered is the one described in the previous section. The fire front evolves in the domain where four thermocouples (T1, T2, T3, T4) are located, after 10 m, 20 m, 30 m, 40 m from the ignition point. The four zones between the thermocouples will be later on called section A,B,C,D.

In order to show a better description of the domain, a schematic is reported in Figure 2.

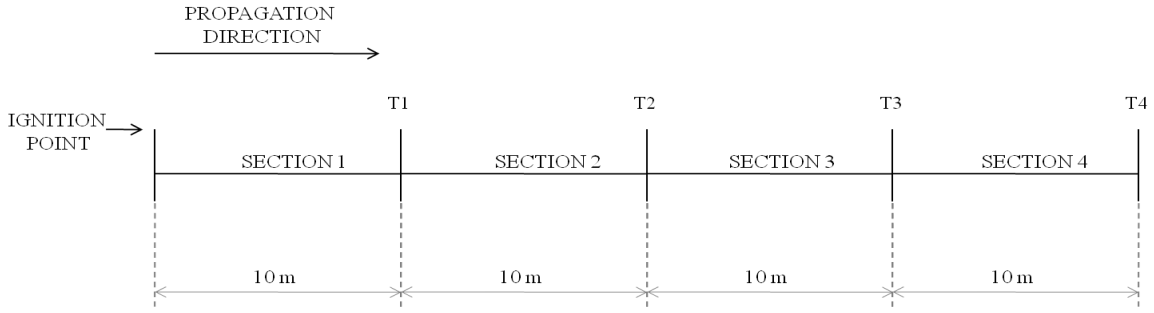

Figure 2. Experimental and model domain

The model is based on energy equation for the fuel array.

$$\rho c \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} - h(T - T_e) - \frac{H}{s}\frac{dM}{dt} + \Phi_{RAD} \quad (1)$$

The mass rate variation, according to Balbi *et al.* (1999) was described using a constant rate of change after reaching the ignition temperature:

$$\frac{dM}{dt} = -\alpha M \quad (2)$$

The terms of the right-hand side of Eq. (1) account for heat transfer due to thermal diffusion, convection ad radiation. A further term appears and accounts for the heat of reaction due to fuel combustion. Fuel properties are different in the four sections of the domain highlighted in Figure 2. In fact both fuel depth and fuel array density are not the same in all the experimental field. In order to obtain results that describes the overall fire behavior the fuel properties used in the model are the averaged ones. Density and specific heat are evaluated considering the fuel array as a porous medium. The convective losses are considered as proportional to the difference between the fuel array temperature and the environmental temperature. The convective losses coefficient was evaluated using experimental data.

Temperature time evolution measured during the experiment was used to evaluate the unknown coefficient in the energy equation. Experimental temperature profile (Figure 3) was investigated and its portion dominated by convective losses (zone 1) was considered for the evaluation of the convective heat transfer coefficient The linearized reaction rate α was estimated considering the portion of temperature evolution (zone 2) mainly affected by the heat release due to combustion The radiative term was estimated using Stefan-Boltzman law (Incropera, Dewitt 1996). The emissivity has been evaluated as described in Pastor *et al*. (2002) ,while the height of the flame is also experimentally determined. The considered radiative term is:

$$\Phi_{RAD} = r F_{ij} \varepsilon \sigma T^4 \quad (3)$$

where r is evaluated using the same curve used for evaluating convective losses coefficient and mass losses coefficient. In this case the zone C of Figure 3 has been considered, when the arriving flame front exchange radiative thermal flux to thermocouple.

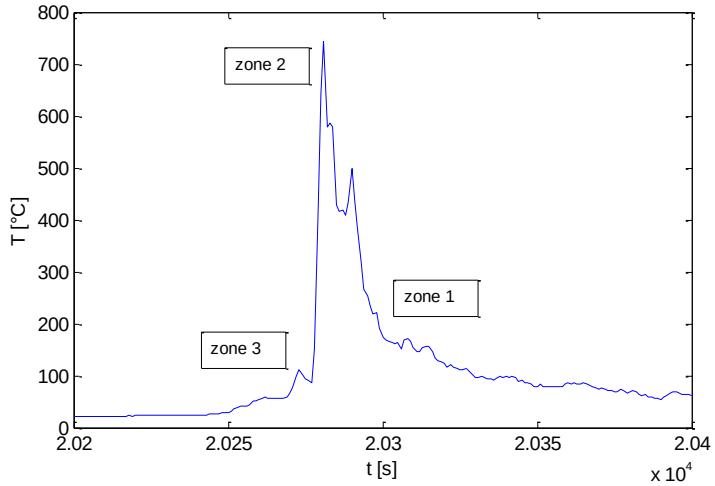

Figure 3. Temperature evolution collected to a thermocouple

Equation (1) is discretized using an implicit finite difference scheme (Ferziger, Peric 2002) and the resulting system of ordinary differential equation is:

$$\boldsymbol{C\dot{T} = AT + DT^4 + f} \tag{4}$$

This is the full-model that is solved in order to obtain the temperature distribution. Time integration is performed using the implicit backward Euler method while non-linearity due to radiative heat transfer are treated through the Newton-Raphson algorithm. Dirichlet boundary conditions were imposed at the left and right boundaries. As initial condition in the first 0.4 m of the domain a sinusoidal temperature distribution with a maximum of 700 K was imposed, in order to represent the fuel ignition. The initial mass per unit area is 0.39 kg/m^2 everywhere.

2.3. POD reduced model

POD is a technique able to capture the main characteristics of a system behavior with a dimensionally reduced model (Sirovich 1987). The reduction is carried out using a collection of sampled values of the considered field called snapshots. The snapshots at M different time are collected in the so-called *snapshot matrix S*, a N x M matrix. Snapshots can be obtained from experiments or simulations. The snapshots matrix can be expressed as:

$$\boldsymbol{S \cong Ba} \tag{5}$$

where $a \in \mathbb{R}^{KxM}$ is the coefficients matrix and $B \in \mathbb{R}^{NxK}$ is the matrix of the truncated basis. N and K are respectively the orders of full model and reduced model.
In order to find the K best "modes" to describes the system an eigenvalue problem is solved. For details refer Bialeki *et al*. (2005). Using this procedure, the system is solved for the temporal contribution of the temperature field α because the space contribution is included yet in the matrix B.

$$\boldsymbol{T = B\alpha} \tag{6}$$

Using (6) and multiplying by B^T equation (4) becomes:

$$\boldsymbol{\tilde{C}\dot{\alpha} = \tilde{A}\alpha + B^T D(B\alpha)^4 + B^T f} \tag{7}$$

where

$$\widetilde{\boldsymbol{C}} = \boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{C}\boldsymbol{B} \text{ and } \widetilde{\boldsymbol{A}} = \boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{A}\boldsymbol{B} \quad (8)$$

The full model dimension is 4000. The POD basis chosen for the analyzed problem are 160, therefore the dimension is reduce to the 5% of the initial one.

3. Results

Temperature distribution at different instants of time obtained with the full model is detailed in Figure 4. It shows that fire rapidly reaches a shape that does not change anymore in time; this condition will be later on called pseudo steady state. The temperature profile in the combustion region varies only in the first 500 s then pure propagation occurs. As a result, the rate of spread is constant.

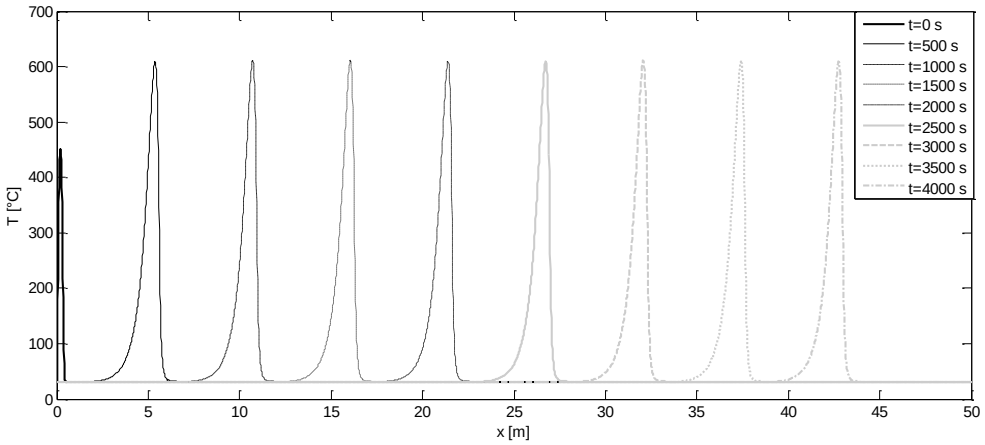

Figure 4. Temperature distribution at different time -Full model

In order to better compare the results obtained with the full model and the experimental ones the average rate of spread in the sections B, C, D of the experiment of the experimental fire front is reported in Figure 5. Figure 5 indicates that a good agreement between experimental ROS and the rate of spread calculated with the proposed model is achieved.

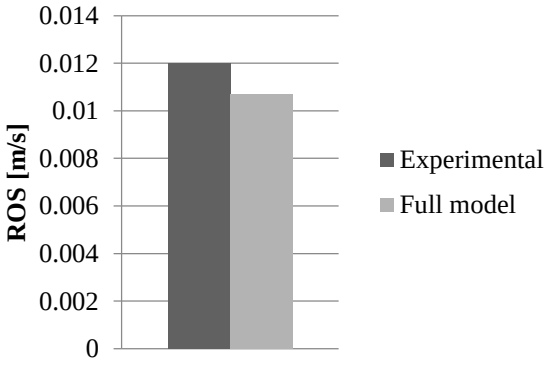

Figure 5. ROS mean value comparison

In Figure 6 the experimental temperature time evolution recorded by thermocouples T1, T2, T3, T4, is reported, which provides information on experimental fire propagation. The evolution of the experimental fire is not constant during time. In fact the front reaches the first thermocouple (section A) after 1000 s, the second after 2000 s, the third after 2800 and the fourth after 3700 s. Therefore ,

since the thermocouples are equidistant it is clear that the ROS in the sections of the domain are very different. In Figure 6 the temperature time evolution predicted using the full model is also reported. The results indicate that a good agreement between experimental data and numerical simulation is achieve, although the model slightly under-predicts the fire front velocity in the first half of the domain. Numerical predictions obtained using the POD model are also compared with experimental data and numerical results of the full model. POD model and the full one are in good agreement and also the maximum temperature predicted is very similar. The marginal differences that can be observed in Figure 6 are due to the fact that only 160 autofunctions are used in order to reduce the computational cost. In the case of the described experiment, the computational time required to solve the energy equation using the POD model with a refined grid is about 10% of the full model.

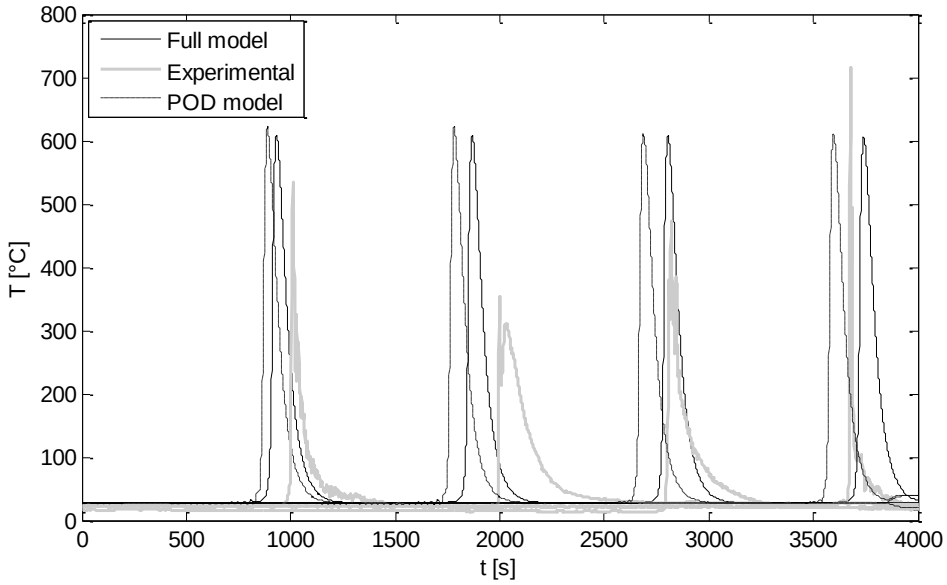

Figure 6. Temperature evolution in the four thermocouple, full model, experimental and POD model

An interesting application of the POD model is the possibility of performing sensitivity or parametric analysis using a reduced model built from a single reference scenario. In particular, it is possible to assess the influence of boundary conditions or model parameters at the expense of small computational costs. In fact the main strength of this approach is the ability to reproduce the behavior of the system with some different characteristics, using snapshots provided to an experiment or simulation executed in other conditions. In this paper the effect of uncertainties in the evaluation of the linearized reaction rate are investigate using the POD model. In particular, a +/- 30% variation of α (Eq. 2) was considered. The resulting variation of ROS are illustrated in Figure 7. The rate of spread evaluated with the full and POD model are very similar; the maximum error obtained is less than 6%. Furthermore the considered case is characterized by a low propagation velocity, therefore the error has a very low value. Figure 7 shows that an higher value of α correspond to a slower propagation, while ROS decrease with smaller value of α.

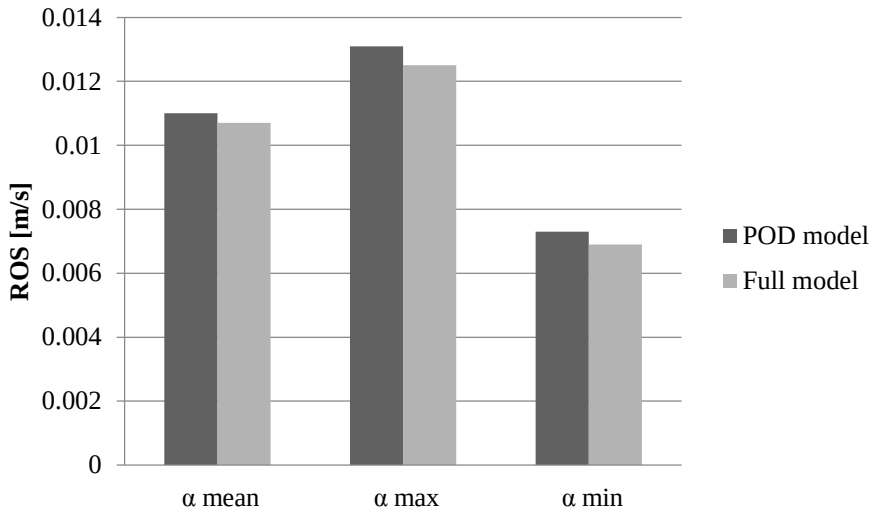

Figure 7. ROS with different mass loss rate coefficient

4. Conclusions

The present paper reports the use of POD in order to reduce the computational time of wildfire models. POD has been applied to a simple 1D problem based on energy conservation equation. Diffusion, convection, radiation, and heat release associated with the mass rate has been considered. Coefficients are determined experimentally. Time integration is evaluated with the implicit backward Euler method and the resulting nonlinear system of equations is solved using a Newton-Raphson algorithm. This is the first paper in literature that deal with the POD reduction applied to forest fire model.
The full model is able to reproduce the pseudo steady-state behavior of the experimental fire. In fact the ROS of the two model are similar. We found that POD model is able to provide appreciative results using only 160 basis. Therefore the dimension of the solved model is 160 while the dimension of the full one is 4000. The computational time to solve the energy equation is reduced to 10% of the full model. This is a good result but in future in order to increase efficiency of non-linear problems solved using POD techniques as the DEIM decomposition can be used.
The snapshots provided to the full model are used in order to simulate fire evolution with two different value of mass loss rate coefficient. Results show that POD model is able to estimate the ROS with good accuracy, even when the equation parameters differ from that used to obtain the snapshots. Errors are below 6% in each the computed cases. Future work will be focused on increasing of performances of this technique and application to more complex and multi dimensional models.

5. Nomenclature

c	specific heat of the fuel array	[kJ/kgK]
F_{ij}	view factor	[-]
h	convective coefficient	[W/m^{3}K]
H	heat content of fuel	[kJ/kg]
k	equivalent conductivity coefficient	[W/mK]
M	mass of fuel per unit of surface	[kg/ m^2]
s	fuel array depth	[m]
r	radiation coefficient	[m^{-1}]

T	temperature of the fuel array	[K]
T_e	temperature of environment	[K]
α	mass loss rate parameter	[s^{-1}]
ρ	density of the fuel array	[kg/m^3]
σ	Stefan-Boltzman constant	[$Wm^{-2}K^{-4}$]
Φ_{RAD}	radiative heat flux	[W/ m^3]

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